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COMMENT

Comments on real-space renormalisation group for directed lattice animals

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Abstract. We carefully analyse the RG results of directed site lattice animals on a square lattice obtained from a new RSRG method proposed by us. Here, asymptotic forms for $\nu_{\parallel}(b+1, b)$ and $\nu_{\perp}(b+1, b)$, the values obtained from cell-to-cell transformation of linear size $b+1$ to b, are proposed in the large b limit. By fitting the data, we find $\nu_{\parallel} = 0.796$ and $\nu_{\perp} = 0.507$. These results are in very good agreement with the known values.

Recently, we proposed a new real-space renormalisation group method to treat systems with directionally dependent critical behaviour (Zhang and Yang 1984, hereafter referred to as ZY). In this method, two effective lengths $S_{\parallel}(b)$ and $S_{\perp}(b)$ are introduced for a given cell of linear size b. The renormalised lattice is constructed from these effective lengths and is deformed from the original lattice. It is the anisotropic rescaling of these effective lengths which give the anisotropic exponents $\nu_{\parallel}$ and $\nu_{\perp}$ characterising the directed systems.

For the case of directed self-avoiding walks (DSAW), ZY have shown in two dimensions that, in the $b \to \infty$ limit, this method is capable of reproducing the correct values of $\nu_{\parallel}$ and $\nu_{\perp}$, i.e. $\nu_{\parallel} = 1$ and $\nu_{\perp} = \frac{1}{2}$ (Redner and Majid 1983), using either the first- or second-moment definition of $S_{\parallel}(b)$ and $S_{\perp}(b)$ (see equations (2)-(5) of ZY). Applying this method to directed lattice animals, reasonably good values of $\nu_{\parallel}$ and $\nu_{\perp}$ have been obtained for the case of bond animals on a square lattice (ZY). The value of $\nu_{\perp}$ is known to be exactly $\frac{1}{2}$ (Cardy 1982 and Dhar 1983). However, there are discrepancies between the estimated values of $\nu_{\parallel}$. Redner and Yang (1982) suggested $\nu_{\parallel} = 0.800$. Breuer and Janssen (1982) proposed $\nu_{\parallel} = 0.7933$ while Nadal *et al* (1982) conjectured $\nu_{\parallel} = \frac{9}{11}$. For the case of site animals, ZY have done the RSRG calculations, using both the first- and second-moment definitions of $S_{\parallel}(b)$ and $S_{\perp}(b)$, up to $b = 6$ and $b = 5$ respectively for the R_0 and R_1 rules (Reynolds *et al* 1980). For all the four cases considered, the results of $\nu_{\parallel}(b+1, b)$ and $\nu_{\perp}(b+1, b)$, the values obtained from cell-to-cell transformation of linear size $b+1$ to b, seem to approach the known values in the $b \to \infty$ limit. To give a more convincing test of the method we analyse the RG results more carefully in order to obtain quantitative results.

We have extended the RG calculations for directed site lattice animals in two dimension up to $b = 6$ in the case of the R_1 rule. The results for $\nu_{\parallel}$ and $\nu_{\perp}$ are given in table 1 for both first- and second-moment definitions of $S_{\parallel}(b)$ and $S_{\perp}(b)$. Now we make the following plausible assumptions. For the cell-to-site transformation, the effective lengths $S_{\parallel}(b)$, $S_{\perp}(b)$ and the eigenvalue $\lambda(b)$ of the recursion relation at the

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Table 1. Directed site lattice animals. Results of $\nu_\parallel$ and $\nu_\perp$ for the rescaling of a cell of linear size b to b' on a square lattice using R_1 rule. $\nu_\parallel^{(2)}$ and $\nu_\perp^{(2)}$ are the results obtained from the second-moment definitions of $S_\parallel(b)$ and $S_\perp(b)$.

	b \ b'	2	3	4	5
$\nu_\parallel$	3	1.2934			
	4	1.2068	1.0836		
	5	1.1569	1.0472	1.0000	
	6	1.1237	1.0230	0.9797	0.9548
$\nu_\parallel^{(2)}$	3	1.2493			
	4	1.1715	1.0607		
	5	1.1263	1.0274	0.9842	
	6	1.0962	1.0052	0.9655	0.9425
$\nu_\perp$	3	1.0837			
	4	0.9976	0.8741		
	5	0.9466	0.8354	0.7848	
	6	0.9121	0.8090	0.7620	0.7339
$\nu_\perp^{(2)}$	3	1.0745			
	4	0.9871	0.8618		
	5	0.9367	0.8249	0.7766	
	6	0.9028	0.7997	0.7548	0.7279

fixed point would behave, in the large b limit, like

$$S_\parallel(b) \simeq B_1 b(1 + B_2 b^{-y}) \tag{1}$$

$$S_\perp(b) \simeq C_1 b^{1/\theta}(1 + C_2 b^{-z}) \tag{2}$$

$$\lambda(b) \simeq A_1 b^{1/\nu_\parallel}(1 + A_2 b^{-x}) \tag{3}$$

where $\theta = \nu_\parallel/\nu_\perp$. The reasons that we propose (1)-(3) are the following. In the $b \to \infty$ limit, we certainly have $S_\parallel(b) \sim b$; this gives (1). The exponents $1/\theta$ and $1/\nu_\parallel$ in (2) and (3) are to ensure that correct values of $\nu_\perp$ and $\nu_\parallel$ are approached as $b \to \infty$. The exponents x, y and z are due to the finite size effect. When b is large, the fixed point of the RG transformation is very close to the correct critical point and differs from it by a small finite size correction (Reynolds *et al* 1980). To the first approximation, we can consider the coefficients A_1, A_2, B_1, B_2, C_1 and C_2 as constants and any b dependence in these coefficients gives higher-order corrections to (1)-(3). Thus (1)-(3) are independent of the particular fixed point $K^*(b)$ and can be used for the cell-to-cell transformation. Equations (1)-(3) are indeed the correct expressions for the case of DSAW (ZY).

Using (1)-(3), the cell-to-cell transformation, in the large b limit, gives

$$\nu_\parallel^{-1}(b+1, b) = \frac{\ln[\lambda(b+1)/\lambda(b)]}{\ln[S_\parallel(b+1)/S_\parallel(b)]} \simeq \nu_\parallel^{-1} - A_2 x b^{-x} + (B_2 y/\nu_\parallel) b^{-y} \tag{4}$$

and

$$\nu_\perp^{-1}(b+1, b) = \frac{\ln[\lambda(b+1)/\lambda(b)]}{\ln[S_\perp(b+1)/S_\perp(b)]} \simeq \nu_\perp^{-1} - \theta[A_2 x b^{-x}(C_2 z/\nu_\perp) b^{-z}]. \tag{5}$$

From (4) and (5), to the leading corrections, we can fit our cell-to-cell ($b+1$ to b) RG results to the following form both for $\nu_\parallel$ and $\nu_\perp$.

$$\nu_i^{-1}(b+1, b) = \nu_i^{-1} - a_i b^{-c_i}; \qquad i = \parallel \text{ or } \perp. \tag{6}$$

Fitting the data in table 2 of ZY to (6), we find, for the case of the R_0 rule, $\nu_\parallel = 0.816$, $\nu_\parallel^{(2)} = 0.816$, $\nu_\perp = 0.490$ and $\nu_\perp^{(2)} = 0.522$. Here $\nu_\parallel^{(2)}$ and $\nu_\perp^{(2)}$ are the results obtained from the second-moment definitions of $S_\parallel(b)$ and $S_\perp(b)$. Fitting the data in table 1 of this comment to (6), we find, for the case of the R_1 rule, $\nu_\parallel = 0.797$, $\nu_\parallel^{(2)} = 0.796$, $\nu_\perp = 0.488$ and $\nu_\perp^{(2)} = 0.524$. We would expect that more reliable results can be obtained by fitting four sets of data simultaneously to (6). By doing so, we find $\nu_\parallel = 0.796$ and $\nu_\perp = 0.507$. These results are in very good agreement with the known values.

This simple fitting procedure (6) cannot be used in the case of directed bond lattice animals. In this case, very good values of $\nu_\parallel$ and $\nu_\perp$ are already obtained even for small values of b (ZY). This fact might suggest that the coefficients a_i in (6) are small and more terms should be involved in (6) for fitting. Fluctuations in the values of $\nu_\parallel(b+1, b)$ for small values of b (ZY) also make the simple asymptotic form (6) invalid for fitting.

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References

Breuer N and Janssen H K 1982 *Z. Phys.* B **48** 347
Cardy J L 1982 *J. Phys. A: Math. Gen.* **15** L593
Dhar D 1983 *Phys. Rev. Lett.* **51** 853
Nadal J P, Derrida B and Vannimenus 1982 *J. Physique* **43** 1561
Redner S and Majid I 1983 *J. Phys. A: Math. Gen.* **16** L307
Redner S and Yang Z R 1982 *J. Phys. A: Math. Gen.* **15** L177
Reynolds P J, Stanley H E and Klein W 1980 *Phys. Rev.* B **21** 1223
Zhang Z Q and Yang Y S 1984 *J. Phys. A: Math. Gen.* **17** 1267